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Aerobee-Hi (AJ11-6), Fin Airloads

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Enclosures: (1) Figure 1 - Sketch showing contours of constant pressure difference between the upper and lower surface of a modified Aerobee Fin, $M = 2.0$, $q = 1800\ lb/ft^2$, $\alpha = 4^{\circ}$.

(2) Figure 2 - Same as Figure 1 except $M = 6.0$, $q = 384\ lb/ft^2$.

1. The purpose of this memorandum is to present the method and results of theoretical calculations of air-load distributions over the planform of a modified Aerobee fin. The calculations, which were undertaken to supply information necessary for structural analysis of the modified fin, were made assuming conditions corresponding to flight Mach numbers of 2 and 6 and an angle of attack of 4°. It was believed that the condition at $M = 2$ would show the fin load distribution near maximum fin loading, and that the condition at $M = 6$ might be critical because of the high fin temperatures at that Mach number. The results of the calculations are shown in Figures 1 and 2 as contours of constant pressure difference between corresponding points on the upper and lower surface of the fin.

2. Since an exact solution for the problem of determining fin air load distributions in the presence of the body is prohibitively complex, if not impossible, using known methods, it was necessary to make certain simplifying approximations. It is believed, however, that the calculations described in this memorandum resulted in load distributions whose accuracy is consistent with the accuracy required by the structural analysis for which the loads were intended. The general method of solution was to separate the complete problem into the following sub-problems.

a. Selection of a fin-body geometry which would adequately represent the actual Aerobee fin, and at the same time permit a moderately simple solution of the problem.

b. Determination of air loading on a flat plate having the planform selected for a. This loading included fin root effects, fin tip effects, and leading-edge sweep effects.

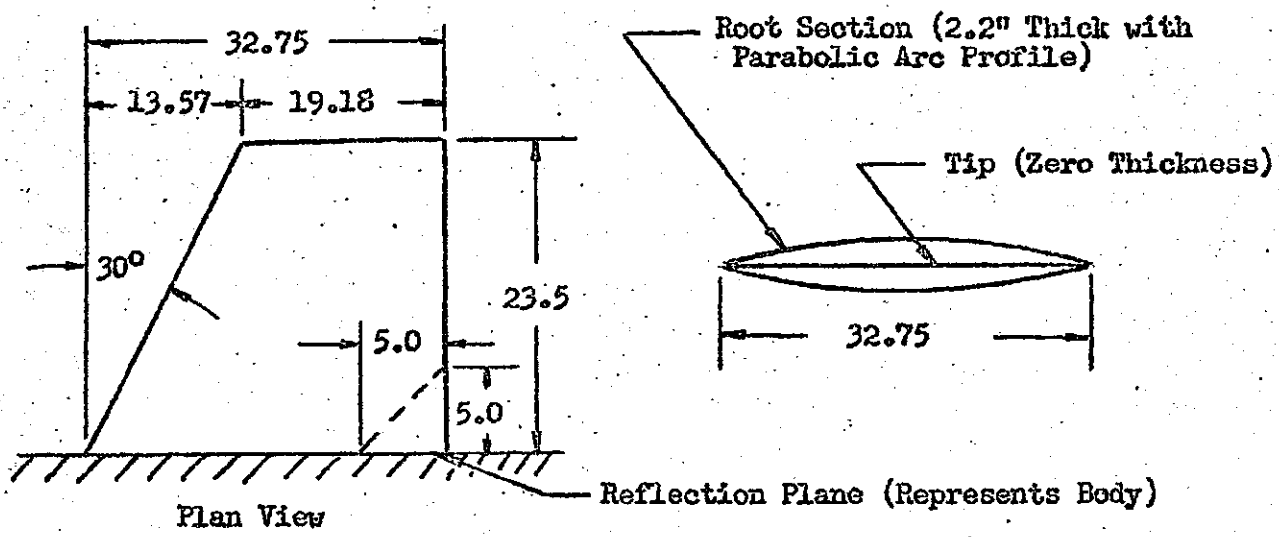
c. Determination of effects of body upwash on the fin load distribution.

d. Determination of effects of fin thickness.

e. Combination of sub-problems b, c, and d to obtain the final air loads shown in Figures 1 and 2.

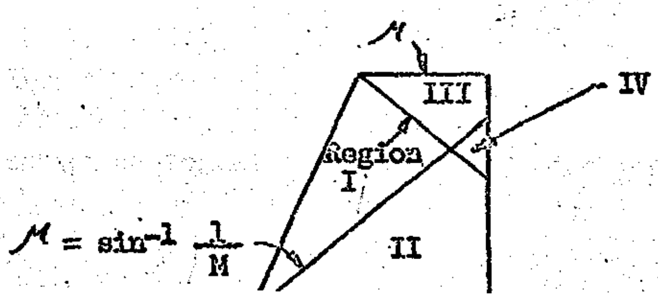
The solutions of the sub-problems are described in the sections following.

3. The following sketch shows the geometry selected as adequately representing the modified Aerobee fin.



The loss of lift near the fin root due to the presence of the body was approximated by treating the body as a reflection plane. (The effect of body upwash was treated separately as described in Section 5). The dashed line near the intersection of the fin trailing edge and body represents a cutout at the trailing edge of the actual fin. On the actual fin, the edge of the cutout is supersonic at M = 1.4, so that there was no effect of misrepresenting the planform on the loading of the remainder of the fin at the Mach numbers selected. The effect of misrepresenting the profile in this region to obtain the simplified geometry was small and was in a direction such that the calculations made using the simplified geometry were conservative in the region which was affected. The effect of misrepresenting the profile in the region ahead of the cutout, and effects of the body boattail, which will be present on some of the actual vehicles using the modified fin, were neglected.

4. The problem of the lift distribution on a fin of zero thickness, which consisted of determining the effect of sweep, tip effects, and variations in loading near the fin root, was solved by finding the load distribution on a flat fin of zero thickness having the planform that was selected in Section 3. The fin was considered to be in a uniform airstream at a small angle of attack. The first step in the solution of this sub-problem was to divide the planform into four separate regions, the fin surface pressures in each region being governed by different equations.



The lines separating the regions are Mach lines through the fin leading edge at the root and tip. It should be noted that Region IV diminishes in size as the Mach number increases. At $M = 2$ all four regions exist, whereas at $M = 6.0$ only regions I, II, and III exist. The equations which were used for finding fin pressures in each of the four regions are presented in the following paragraphs.

A. Fin Loading In Region I

For a flat fin of zero thickness at a small angle of attack in a uniform airstream, uniform loading exists in Region I. Using sweepback theory, the difference in pressure coefficient between corresponding points on the upper and lower surface of the fin is given by: (See Ferri, p. 361).

$$\Delta C_p = \frac{4\alpha}{\sqrt{M^2 - 1}\sqrt{1 - \frac{\tan^2 \phi}{M^2 - 1}}} = \frac{4\alpha}{\sqrt{M^2 - 1 - \tan^2 \phi}}$$

where $\Delta C_p = \frac{P_\ell - P_u}{q}$

$q = \frac{1}{2}\rho V^2$ (dynamic pressure in lb/ft^2)

P_ℓ = Static pressure on fin lower surface (lb/ft^2)

P_u = Static pressure on fin upper surface (lb/ft^2)

ρ = Free stream mass density (slugs/ft^3)

V = Free stream velocity (ft/sec)

M = Free stream Mach number

α = Fin angle of attack (radians)

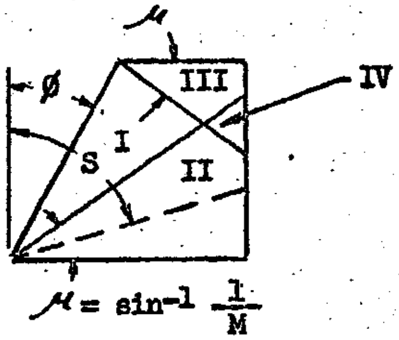
ϕ = Sweepback angle of fin leading edge

B. Fin Loading In Region II

In Region II (still considering the flat fin in a uniform stream) the flow is conical. The surface pressure is constant along each radial line passing through the fin-root leading edge. The difference in pressure coefficient between the upper and lower surface is given by: (See Ferri p. 362)

$$\Delta C_p = \frac{4\alpha}{\sqrt{M^2 - 1 - \tan^2 \phi}}\left(1 - \frac{2}{\pi}\sin^{-1}\sqrt{\frac{\frac{\tan^2 \phi}{M^2 - 1} - \frac{\tan^2 \phi}{\tan^2 S}}{1 - \frac{\tan^2 \phi}{\tan^2 S}}}\right)$$

where $\triangle C_p$, α, M, $\emptyset$, are defined in Section 4, A and S is the sweep of the line in Region II passing through the fin-root leading edge, along which $\triangle C_p$ is to be calculated. See the sketch below.



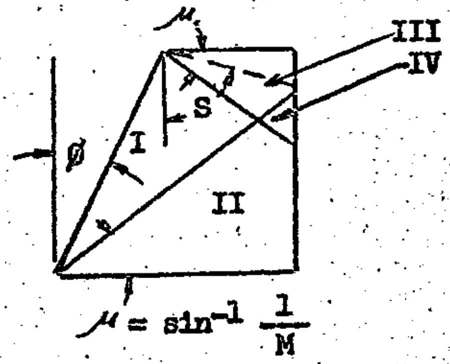
C. Fin Loading in Region III

In Region III, the flow is also conical, the pressures being constant along radial lines passing through the fin-tip leading edge. The pressure distribution in this region differs from that of Region II in that at the fin tip the loading drops to zero. In Region III the difference in pressure coefficient between the upper and lower surface is: (See Douglas Rep. SM 11901 p. 3).

$$\triangle C_p = \frac{4\alpha}{\sqrt{M^2 - 1 - \tan^2 \emptyset}} \left(\frac{1}{\pi} \cos^{-1} \frac{\tan S - \left(\cancel{2}\sqrt{M^2 - 1} + \tan \emptyset\right)}{\tan S + \tan \emptyset} \right)$$

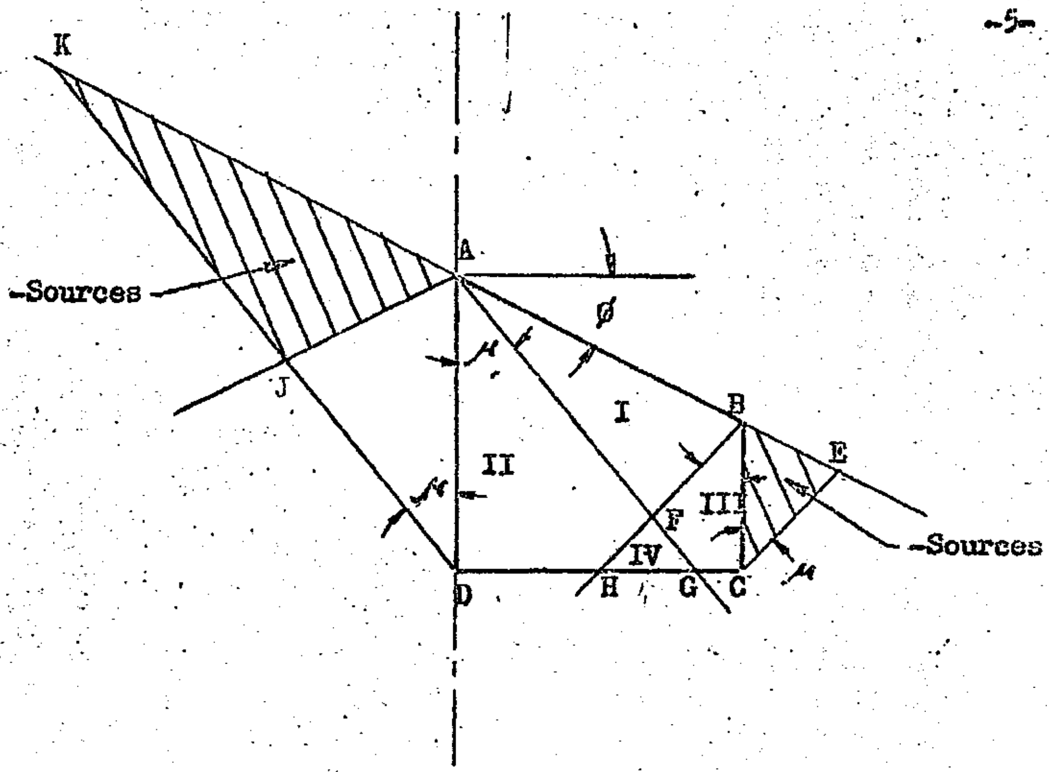
See Ferri pg 379 for $\phi = 0°$ eq 804

where $\triangle C_p$, α, M, $\emptyset$, are defined in Section 4, A and S is the sweep of a line in Region III which passes through the fin-tip leading edge, and along which $\triangle C_p$ is to be calculated. See sketch below.



D. Fin Loading in Region IV

The relation governing the loading in Region IV can be easily developed using the expressions presented in Sections 4 A, B, and C, by the following considerations.



The pressure distribution over one surface of fin ABCD can be considered as resulting from a uniform source distribution over the entire area KECD combined with a non-uniform negative source distribution over each of the areas AKJ and BEC. The two non-uniform negative source distributions are independent of each other and result in the deficiences in lift from that of ABF (Region I) which are experienced by the root and tip regions (ADG and BDH). Since FGH (Region IV) is affected by both of these negative source distributions, the deficiency in lift in Region IV at any point can be expected to equal the combined deficiencies at that point which are associated with regions ADG and BCH.

Following this reasoning, and adding subscripts to the quantities ΔC_p and S, defined earlier, in order to denote the region for which a particular ΔC_p or S pertains, the following relations can be written.

$$\left(1 - \frac{\Delta C_{p_{IV}}}{\Delta C_{p_I}}\right) = \left(1 - \frac{\Delta C_{p_{II}}}{\Delta C_{p_I}}\right) + \left(1 - \frac{\Delta C_{p_{III}}}{\Delta C_{p_I}}\right)$$

solving for $\frac{\Delta C_{p_{IV}}}{\Delta C_{p_I}}$ the result is:

$$\frac{\Delta C_{p_{IV}}}{\Delta C_{p_I}} = \frac{\Delta C_{p_{II}}}{\Delta C_{p_I}} + \frac{\Delta C_{p_{III}}}{\Delta C_{p_I}} - 1$$

See eq 8.24 page 382 of Jones for Tip-Tip interaction for $\beta = 0°$

The point for which $\frac{\Delta C_{p_{IV}}}{\Delta C_{p_I}}$ pertains, lies at the intersection of the two lines along which $\frac{\Delta C_{p_{II}}}{\Delta C_{p_I}}$ and $\frac{\Delta C_{p_{III}}}{\Delta C_{p_I}}$ are calculated.

$\frac{\Delta C_{p_{II}}}{\Delta C_{p_I}}$ and $\frac{\Delta C_{p_{III}}}{\Delta C_{p_I}}$ can be easily determined using equations given in Section 4, A, B, and C. They are:

$$\frac{\Delta C_{p_{II}}}{\Delta C_{p_I}} = \left[1 - \frac{2}{\pi} \sin^{-1} \sqrt{\frac{\dfrac{\tan^2 \phi}{M^2 - 1} - \dfrac{\tan^2 \phi}{\tan^2 S_{II}}}{1 - \dfrac{\tan^2 \phi}{\tan^2 S_{II}}}} \right]$$

and

$$\frac{\Delta C_{p_{III}}}{\Delta C_{p_I}} = \frac{1}{\pi} \cos^{-1} \frac{\tan S_{III} - 2\sqrt{M^2 - 1} + \tan \phi}{\tan S_{III} + \tan \phi}$$

5. Rather than being subjected to a uniform airstream the fins are, in actuality, subjected to a stream of whose angle of attack varies in a spanwise direction because of the upwash due to the presence of the body. Beskin has pointed out that the upflow angle due to a cylindrical body varies spanwise on the horizontal plane of symmetry as

$$\frac{\alpha_\ell}{\alpha} = 1 + \frac{a^2}{r^2}$$

where:

α_ℓ is the local angle of attack at a distance r from the body centerline

a is the body radius

α is the body angle of attack

To obtain the complete solution of a fin subjected to a stream having this variation of local angle of attack would be very tedious even using linearized equations. The method used for the calculations described in this Memorandum consisted merely of using the approximation that the local values of ΔC_p on the fin are directly proportional to the local angles of attack as calculated with the Beskin relation. The body upwash effects then take the form of corrections to the solution of the flat plate airfoil in a uniform stream, the magnitude of the corrections being a function only of the spanwise location. The relation which was used for the present calculations is:

$$\frac{\Delta C_p \text{ corrected for upwash}}{\Delta C_p \text{ without upwash correction}} = 1 + \frac{a^2}{r^2}$$

6. The linearized theory gives the results that the difference in pressure between the upper and lower fin surfaces is independent of fin thickness. However, more nearly exact calculations and experimental measurements indicate that there is a dependence of fin lift distribution on profile shape, the variations of chordwise lift distribution with profile shape increasing as the Mach number becomes larger. It was decided that neglecting the effect of thickness in calculations made for $M = 2.0$ was consistent with the accuracy required by the structural analysis for which the fin loads were to be used. However, at $M = 6.0$ the effect of thickness becomes fairly large, so for completeness, it was considered.

The first step in obtaining the correction for ΔC_p to account for fin thickness at $M = 6.0$ was to determine the chordwise load distribution on a two-dimensional unswept airfoil at an angle of attack of 4° having the profile of the fin root selected in Section 3 using the "shock-expansion" method, (See Ferri, p. 125). The "shock-expansion" method consists of determining the flow conditions at the airfoil leading edge downstream of the leading edge shock wave using two-dimensional, oblique shock theory, and then using the relations for a simple-wave expansion (the flow deflection angle plus the Prandtl-Meyer angle equals a constant) to calculate the flow over the airfoil surfaces between the leading and trailing edges.

From the results of the "shock-expansion" calculations, the ratio of ΔC_p for the fin-root profile to ΔC_p for a flat plate at the same conditions was determined as a function of the percent chord. This ratio was then assumed to vary linearly, at a given percent chord, in a spanwise direction from the value calculated at the root to unity at the tip. The final form of the correction was a plot of the ratio of the local ΔC_p to ΔC_p for a flat plate as a function of percent chord with spanwise station as a parameter.

7. The final results of the calculations described by this Memorandum are shown by contours of constant pressure difference between corresponding points on the upper and lower surfaces of the modified Aerobee fin; the calculations were performed for the following two sets of conditions:

1. $M = 2.0,\ q = 1800\ \text{lb/ft}^2,\ \alpha = 4^\circ$
2. $M = 6.0,\ q = 384\ \text{lb/ft}^2,\ \alpha = 4^\circ$

The values of q (dynamic pressure) were selected corresponding to the above values of Mach number from suitable trajectory calculations. The final contour plots were obtained directly from preliminary plots which showed the spanwise variation of fin loading at several chordwise stations. At any point on the fin the final value of $P_\ell - P_u$ in lb/ft was, in principle, obtained using the following procedure:

a. The "region" of the fin which included the point was determined and a value of ΔC_p for a flat fin of zero thickness was computed using the appropriate equation or equations of Section 4.

b. Knowing the spanwise location of the point, the ΔC_p of a, was corrected for body upwash using the equation presented in Section 5.

c. From the spanwise location and the percent chord of the point, the thickness correction was obtained from the plot described at the end of Section 6. The thickness correction multiplied by the ΔC_p of b, resulted in the final value of ΔC_p for the selected point. The correction for fin thickness was applied only to the calculations made for $M = 6.0$.

d. The final pressure difference in lb/ft^2 at the selected point was obtained by multiplying the ΔC_p of c, by the dynamic pressure in lb/ft^2.

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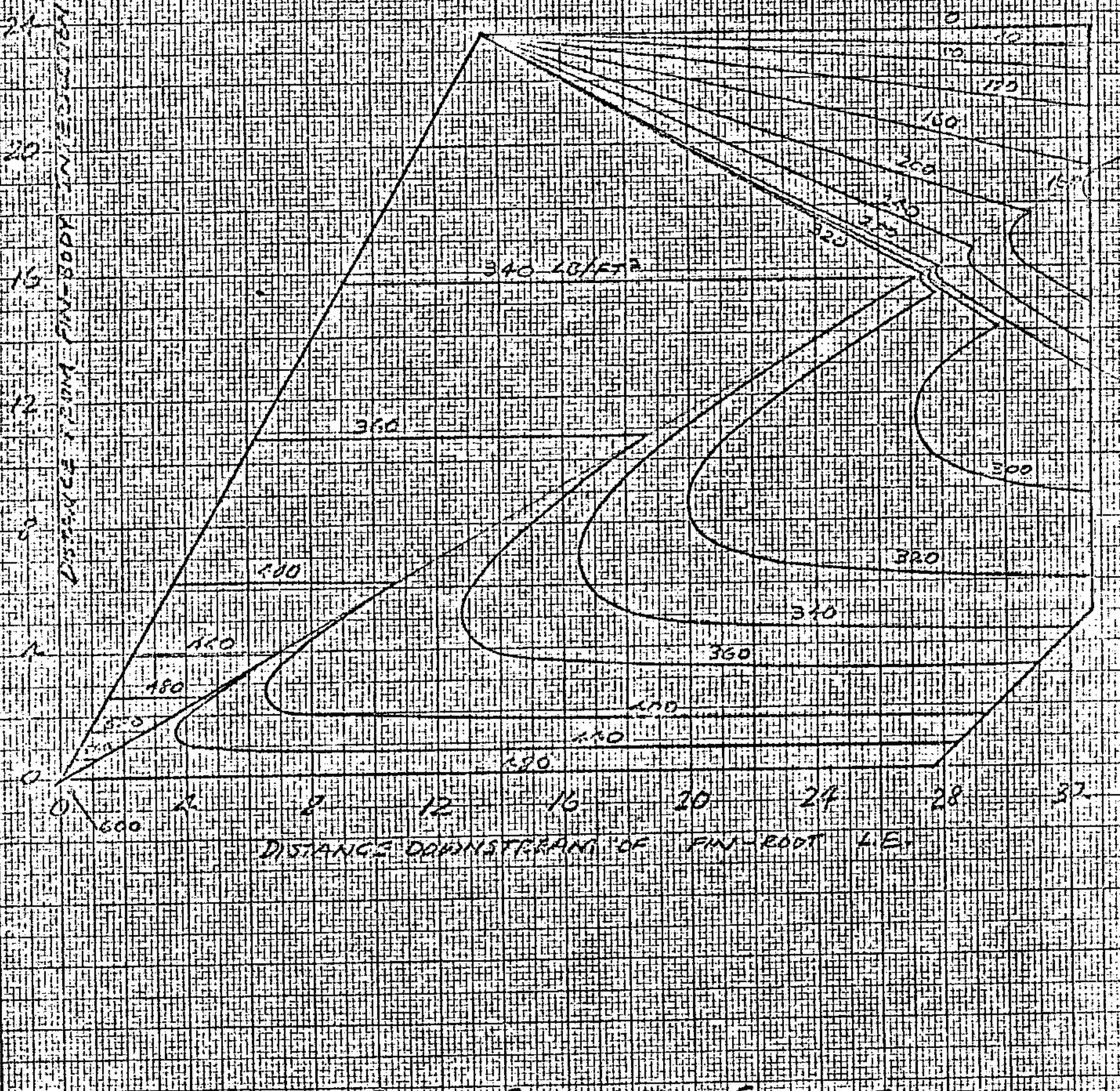

SKETCH SHOWING CONTOURS OF CONSTANT PRESSURE DIFFERENCE BETWEEN THE UPPER AND LOWER SURFACE OF A MODIFIED ABBOBER FIN

$M = 2.0$, $q = 1800$ LB/FT², $\alpha = 4°$

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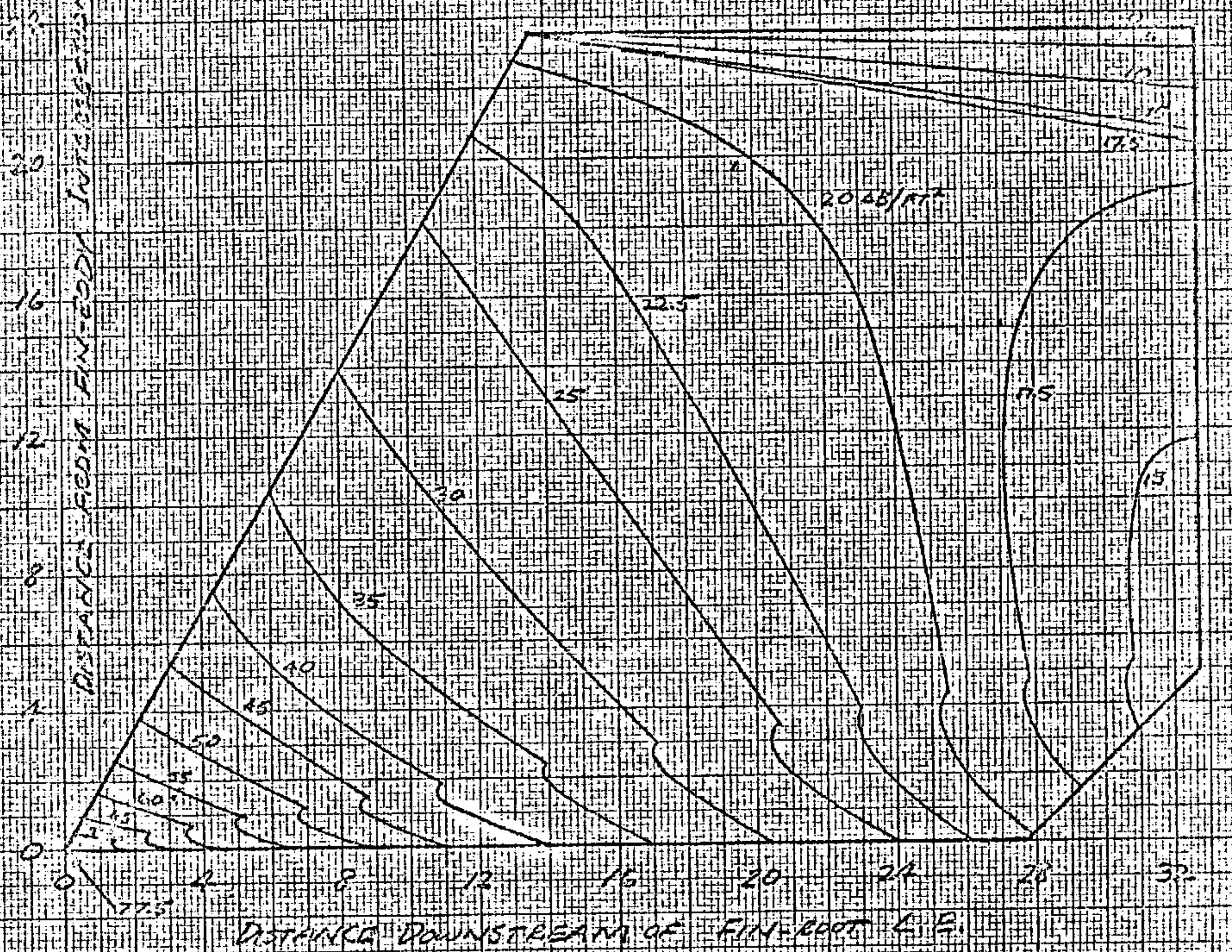
Figure 1



SKETCH SHOWING CONTOURS OF CONSTANT PRESSURE DIFFERENCE BETWEEN THE UPPER AND LOWER SURFACE OF A MODIFIED AEROBEE FIN

$M = 6.0$, $q = 384$ LB/FT², $\alpha = 4°$

Figure 2